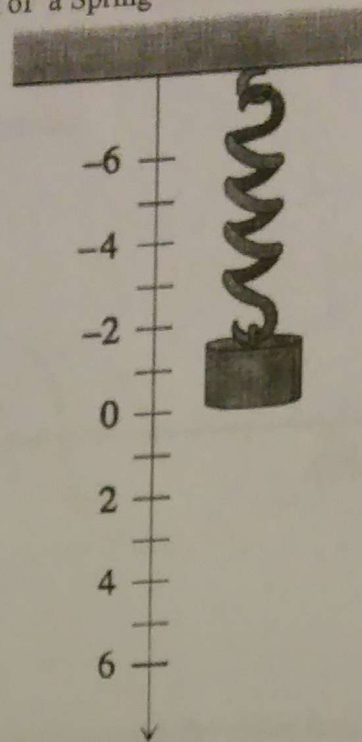
Modeling the Motion of a Spring



Consider a weight attached to a spring that is suspended from a horizontal bar as illustrated in the figure. When the object comes to rest we say it is at "equilibrium" which is labeled 0 on the vertical number line. If you give the weight a push, either up or down, it will start to move and the motion can be modeled by sine and cosine functions. The "stiffness" of the spring and the mass of the object affect how far the object moves from the equilibrium position. The initial velocity and initial position also affect the motion of the spring. (We don't always start at the equilibrium position.)

If we neglect any damping forces (air resistance etc.) then the motion of the spring can be modeled by

$$x(t) = \frac{v_0}{\omega}\sin(\omega t) + x_0\cos(\omega t)$$

where $x(t)$ is the position of the object along the number line at time t. The other quantities are constants: ω is a constant that depends on the stiffness of the spring and the mass of the weight, v_0 is the initial velocity, and x_0 is the initial position of the object.

Model the motion of a weight on a spring:

Suppose a weight is set in motion from a position 3 centimeters below the equilibrium position and with a downward velocity of 4 centimeters per second. (Please note that the vertical number line used for position is "upside down". This is a convention from physics and it means that positions below equilibrium actually correspond to a positive value.) Assume that the spring stiffness and mass of the weight mean that $\omega = 2$ for this system.

Part I

1) Write the function $x(t)$ that gives the position of the weight as a function of time t in seconds. (Your function should consist of a sine term and a cosine term.)

$x(t) = \frac{V_0}{\omega}\sin(\omega t) + X_0\cos(\omega t)$

$\omega = 2$

$V_0 = 4$

$X_0 = 3$

$\frac{4}{2}\sin(2t) + 3\cos(2t)$

$2\sin(2t) + 3\cos(2t)$